Exercise:

Consider two single-layer perceptrons, **A** and **B**, each defined by the inequality:

$$w_0 + w_1x_1 + w_2x_2 \geq 0$$

The weight parameters for each perceptron are as follows:

- **Perceptron A:** $w_0 = 1$, $w_1 = 2$, $w_2 = 1$
- **Perceptron B:** $w_0 = 0$, $w_1 = 2$, $w_2 = 1$

Determine whether **Perceptron A** is more general than **Perceptron B** in terms of classification.

Perceptron a:

$$1 + 2x_1 + x_2 \geq 0$$

which rearranges to:

$$x_2 \geq -2x_1 - 1$$

Perceptron b:

$$0 + 2x_1 + x_2 \geq 0$$

which simplifies to:

$$x_2 \geq -2x_1$$

Comparison:

- Both perceptrons have similar decision boundaries with slopes of **-2**, meaning they are parallel.
- The main difference is in the intercept:
  - **Perceptron a** has an intercept of **-1**.
  - **Perceptron b** has an intercept of **0**.

Since **perceptron a** has a lower intercept, its decision boundary is **below** perceptron b's decision boundary. This means that perceptron a classifies **more points as positive** (i.e., satisfying the inequality) compared to perceptron b.

Conclusion:

Since perceptron a accepts all points that perceptron b does and potentially more, **perceptron a is more general than perceptron b.**

Perceptron Learning Algorithm

Repeat until convergence (or for a fixed number of iterations):

1. **For each training example** $(f_1, f_2, \ldots, f_n)$ with label y:
   - Compute the prediction:
     $$\hat{y} = \text{sign}\left(\sum w_i f_i + b\right)$$
   - If the prediction is incorrect ($\hat{y} \neq y$ or $\hat{y} \cdot y \leq 0$):
     - **Update each weight** w_i:
       $$w_i = w_i + f_i \cdot y$$
     - **Update the bias** b:
       $$b = b + y$$

Exercise 1: Implement the Perceptron Algorithm

Implement the **Perceptron Learning Algorithm** for a binary classification problem.

Steps:

1. Initialize weights and bias to zero.
2. Use the following dataset:

x_1	x_2	Label y
1	1	1
-1	-1	-1
2	-2	-1
-2	2	1

3. Apply the Perceptron update rule with a learning rate $\eta = 1$.
4. Run for **5 iterations** and print the updated weights after each iteration.
5. Plot the decision boundary after training.

Exercise 2: Find the Decision Boundary

A perceptron is trained using the **weight vector** $w = (2, -1)$ and **bias** $b = 3$.

1. Write the decision boundary equation in the form:
$$x_2 = mx_1 + c$$
2. Plot the decision boundary on a 2D plane.
3. Classify the following points using the perceptron decision rule and determine whether they belong to class +1 or -1:
   - $(2, 1)$
   - $(0, 3)$
   - $(-1, -2)$

Exercise 3: Weight Updates in the Perceptron

A perceptron is initialized with **weights** $w_1 = 0, w_2 = -1$, and **bias** $b = 0$. A new training example $(x_1, x_2) = (2, 3)$ with label $y = 1$ is presented.

1. Compute the initial prediction $\hat{y}$.
2. If the perceptron misclassifies the point, update the weights and bias using the perceptron learning rule:
$$w_i = w_i + y \cdot x_i$$
$$b = b + y$$
3. Repeat step 2 for the points $(1, -1)$ with label $y = -1$ and $(0, 2)$ with label $y = 1$.
4. Show the updated weights and bias after each step.

A perceptron is trained to classify points in a 2D space using the decision rule:

$$w_0 + w_1 x_1 + w_2 x_2 \geq 0$$

Given the initial weights:

- $w_0 = -2$
- $w_1 = 3$
- $w_2 = 1$

1. Find the equation of the decision boundary.
2. Classify the points $(2, 1)$, $(-1, 4)$, and $(0, 0)$ based on the perceptron's decision rule.
3. Suppose the perceptron misclassifies the point $(2, 1)$, and the correct class label should be 1. Using a learning rate of $\eta = 0.1$, update the perceptron's weights using the perceptron learning rule.
4. Write the new decision boundary equation after the weight update.